Domain structure of superconducting ferromagnets

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In superconducting ferromagnets the equilibrium domain structure is absent in the Meissner state, but appears in the spontaneous vortex phase (the mixed state in zero external magnetic field), though with a period, which can essentially exceed that in normal ferromagnets. Metastable domain walls are possible even in the Meissner state. The domain walls create magnetostatic fields near the sample surface, which can be used for experimental detection of domain walls.

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Recently there has been a growing interest to materials, in which superconductivity and ferromagnetism coexist [1–5]. A number of unusual phenomena and structures have been predicted and observed, spontaneous vortex phase as an example [6,7]. But the theory mostly addressed macroscopically uniform structures, whereas ferromagnetic materials, even ideally uniform, inevitably have a domain structure, which is a ground-state property of ferromagnets. So a further progress in studying materials with coexisting ferromagnetism and superconductivity requires an analysis of the domain structure. The present work is the first step in this direction.

An object of the study is a material, in which the magnetic transition occurs earlier, i.e. at a higher temperature, than the superconductivity onset. This was called "superconducting ferromagnet" [7], in contrast to "ferromagnetic (or magnetic) superconductors" where the superconductivity sets in *before* the magnetic transition, which have been studied mostly in the past [8]. Competition of ferromagnetism and superconductivity may result in various structures with the magnetic moment rotating in space (spiral structures, cryptoferromagnetism and so on). This also can be considered as a "domain structure", but with a period determined by intrinsic properties of materials. However, our goal is the domain structure due to magnetostatic fields generated by nonzero average bulk magnetization $\vec{M}$. In this case the domain size depends on a sample size. We shall consider type-II superconductivity, bearing in mind ruthenocuprates [1,4,5], which are type II high-T_c superconductors.

Before analyzing the domain structure it is useful to summarize the magnetic properties of a single-domain superconducting ferromagnet. The total free energy of the superconducting ferromagnet can be written as [7]

$$F(\vec{M},\vec{B}) = f_E + K + \frac{(\vec{B}-4\pi\vec{M})^2}{8\pi} + \frac{2\pi\lambda^2}{c^2}j_s^2 \ , \qquad (1)$$

where λ is the London penetration depth and $\vec{B}$ is the magnetic induction. The energy $f_E(M,\nabla\vec{M})$ is the exchange energy, which depends on the absolute value of M and on gradients of $\vec{M}$. As a rule [9], in magnetic materials this is the largest energy, which fixes M. The anisotropy energy $K(\vec{M}/M)$ is smaller and depends on the direction of $\vec{M}$. We shall consider a stripe magnetic structure, which is possible only if K essentially exceeds the magnetostatic energy $\sim M^2$ [9]. The latter is determined by the magnetic field $\vec{H} = \vec{B} - 4\pi\vec{M}$ [the third term in Eq. (1)]. The expression Eq. (1) includes also the kinetic energy related to the superconducting current

$$\vec{j}_s = \frac{c\Phi_0}{8\pi^2\lambda^2}\left(\vec{\nabla}\varphi - \frac{2\pi\vec{A}}{\Phi_0}\right) \ , \qquad (2)$$

where Φ_0 is the magnetic-flux quantum, φ is the phase of the superconducting order parameter, and the vector potential $\vec{A}$ determines the magnetic induction $\vec{B} = \vec{\nabla}\times\vec{A}$. The kinetic energy of superconducting currents is absent in a normal ferromagnet.

Minimization of the energy with respect to the vector potential $\vec{A}$ yields the Maxwell equation

$$\frac{4\pi}{c}\vec{j}_s = \vec{\nabla}\times(\vec{B}-4\pi\vec{M}) = \vec{\nabla}\times\vec{B} \ . \qquad (3)$$

Together with the equation

$$\vec{\nabla}\times\vec{j}_s = -\frac{c}{4\pi\lambda^2}\vec{B} \qquad (4)$$

this yields the London equation which determines $\vec{B}$:

$$\lambda^2\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] + \vec{B} = 0 \ . \qquad (5)$$

Here we took into account that $\vec{\nabla}\times\vec{M} = 0$ inside domains. In contrast to Ref. [7], we neglect the differential susceptibility (M does not depend on a magnetic field), which renormalizes the London penetration depth.

These equations and the boundary conditions at the sample boundary (continuity of the tangential component of $\vec{H}$ and of the normal component of $\vec{B}$) yield the distribution of $\vec{B}$ and $\vec{H}$. This distribution is shown in Fig. 1 for the case of $\vec{M}$ parallel to the sample boundary and for zero external magnetic field. The magnetic induction and the related magnetic flux exist only in the layer of the thickness λ. Meissner currents in this layer screen the internal field $4\pi\vec{M}$, as well as they screen the external magnetic field in a nonmagnetic superconductor.

Let us consider now the mixed state of the superconducting ferromagnetic, in which vortices (magnetic fluxons) are present in the bulk. Since ferromagnetism does

not affect the London equation (5), one expect the same magnetic-induction distribution in the mixed state as for nonmagnetic type II superconductors [7], and the free energy is given by

$$F_m(\vec{M},\vec{B}) = f_E + K + 2\pi M^2 - \vec{B}\cdot\vec{M} + F_0(B) \ , \quad (6)$$

where $F_0(B)$ is the free energy of a nonmagnetic type II superconductor, and $\vec{B}$ now is the magnetic induction averaged over the vortex-array cell. The energy $F_0(B)$ contains both the magnetic energy $B^2/8\pi$ and the kinetic energy of the superconducting currents inside the vortex cell. Determining the magnetic field

$$\vec{H} = 4\pi\frac{\partial F_m}{\partial \vec{B}} = 4\pi\frac{\partial F_0}{\partial \vec{B}} - 4\pi\vec{M} \ , \quad (7)$$

we see that the magnetization curve of a superconducting ferromagnet is described by $B = B_0(|\vec{H} + 4\pi\vec{M}|)$ where $B_0(H)$ is the equilibrium magnetization curve for a nonmagnetic type II superconductor [7] (Fig. 2a). Note that in this relation the magnetic field $\vec{H}$ has a different physical meaning from that used in the Meissner state. For the Meissner state we introduced $\vec{H} = \vec{B} - 4\pi\vec{M}$, where the moment $\vec{M}$ originates from "molecular" currents responsible for ferromagnetism, the superconducting currents being treated as external currents. In the mixed state, which is considered now, it is more convenient to define the magnetic field as $\vec{H} = \vec{B} - 4\pi(\vec{M} + \vec{M}_s)$, i.e. the definition includes also the diamagnetic moment $\vec{M}_s = (\vec{B}_0 - \vec{H})/4\pi$ of the superconducting currents circulating around vortex lines in the mixed state. Thus these currents are treated in the same manner as molecular currents responsible for ferromagnetism.

Figure 2b shows that in a superconducting ferromagnet the Meissner state ($B = 0$) exists until $H + 4\pi M < H_{c1}$, where $H_{c1} = (\Phi_0/\lambda^2)\ln(\lambda/\xi)$ is the lower critical field in a nonmagnetic superconductor and ξ is the coherence length, which determines the vortex core size. So ferromagnetism decreases the lower critical field $\tilde{H}_{c1} = H_{c1} - 4\pi M$. If $4\pi M > H_{c1}$, the Meissner state is absent (Fig. 2c) and the superconducting ferromagnet is in the mixed state with vortices penetrating into it even in zero external field $H = 0$. This is *spontaneous vortex phase* with nonzero magnetic induction $B = B_0(4\pi M)$ in the bulk.

Now let us consider formation of the domain structure in the standard geometry [9]: a slab of the thickness d along the anisotropy easy axis y and infinite in directions of the axes x and z (Fig. 3). We start from a normal ferromagnet. In the absence of an external magnetic field the average magnetic induction inside the slab must vanish. Therefore, $B = 0$ in a single-domain structure (Fig. 3a), and there exists an uniform magnetostatic field $\vec{H} = -4\pi\vec{M}$ in the entire sample, an analog of the electrostatic field in a charged plane capacitor. This results in a high magnetostatic energy $H^2/8\pi \sim M^2$. However, the domain structure with period l ($l \ll d$) suppresses this energy in the domain bulk: $\vec{H} \approx 0$ and $\vec{B} = -4\pi\vec{M}$, except for the area $\sim l^2$ near the sample boundary (Fig. 3b). But the average induction still vanishes, since $\vec{M}$ changes its sign from a domain to a domain. For the stripe structure one can solve the equations of magnetostatics, $\vec{\nabla}\times\vec{H} = 0$ and $\vec{\nabla}\cdot\vec{H} = 4\pi\rho_M$, exactly [9,10]. Here $\rho_M = -\vec{\nabla}\cdot\vec{M}$ is the magnetic charge. The magnetostatic energy per unit volume of the slab is

$$E_s = 0.852 M^2 l^2 \times \frac{1}{ld} = 0.852 M^2 \frac{l}{d} \ . \quad (8)$$

This energy is by a factor l/d less than the magnetostatic energy in a single-domain structure. However, the domain walls increase the energy. The energy of one domain wall (per unit length along the slab) is $\alpha K\delta d$, where δ is the wall thickness and the numerical factor α depends on the detailed definition of K and δ. Its specification is not essential for the present analysis. The domain-wall energy per unit volume of the sample is

$$E_w = \alpha K d\delta \times \frac{1}{ld} = \alpha K\frac{\delta}{l} \ . \quad (9)$$

The equilibrium value of the period l is determined by minimization of the energy $E_w + E_s$ [9]:

$$l = \sqrt{\frac{\alpha K}{0.852 M^2}\delta d} \ . \quad (10)$$

Let us return back to a superconducting ferromagnet. In the Meissner state the magnetic induction must vanish in the bulk, which is compatible only with the single-domain structure. Thus *the equilibrium domain structure is impossible in the Meissner state*. However, domains with the changing direction of $\vec{M}$ can appear in the spontaneous vortex phase with nonzero $B = B_0(4\pi M)$. Like in a normal ferromagnet, the magnetic flux $\propto B$ in domains should produce the magnetostatic fields in the area $\sim l^2$, but in a superconducting ferromagnet these fields are by the factor $B_0(4\pi M)/4\pi M$ smaller. We can take it into account introducing the effective magnetization $\tilde{M} = B_0(4\pi M)/4\pi$. Then the period of the domain structure is given by Eq. (10), where M must be replaced with $\tilde{M}$. In the limit of large $4\pi M \gg H_{c1}$, one has $\tilde{M} \to M$ and the effect of superconductivity on the domain structure vanishes. In the opposite limit of small M, when $4\pi M \to H_{c1}$, $\tilde{M}$ vanishes and the period l becomes infinite, as it should be in the Meissner state $4\pi M < H_{c1}$. However, this calculation of l assumes that the penetration of the magnetostatic field into a superconducting ferromagnet is similar to the penetration into a normal ferromagnet. The assumption is correct if rigidity of the vortex array is negligible and the effective penetration depth is infinite. We can also consider the opposite limit of a very rigid vortex array, when the magnetostatic fields penetrate only into the layer of the

thickness λ. If $\lambda \ll l$, the penetration of the magnetic flux into a superconductor becomes insignificant. This increases the magnetic fields outside the sample, as well as the total magnetostatic energy, by a factor of 2, while the correspondingly period l decreases by a factor of $\sqrt{2}$ (Fig. 3c), in analogy with the effect of a superconducting substrate on a domain size in a ferromagnetic slab [10]. Thus avoiding a detailed analysis of the vortex and field pattern in the domains close to the sample border we lose only a numerical factor of not more than $\sqrt{2}$. In any case, superconductivity, which coexists with ferromagnetism in the same bulk, always increases the domain size, in contrast to the superconductor-ferromagnet bilayer, where superconductivity shrinks ferromagnetic domains [10].

The absence of the equilibrium domain structure in the Meissner state does not rule out a possibility of *metastable* domain walls, as topologically stable planar defects. Domains can appear also because of disorder, or grain structure. The structure of the domain wall should be found by solution of the coupled equations of magnetostatics and the London electrodynamics. We restrict ourselves to the simplest case, when the London penetration depth λ essentially exceeds the domain wall thickness δ. This means that at the spatial scales of order δ the domain-wall structure is governed by large energies [the exchange energy and the anisotropy energy, see Eq. (1)] and is not affected by the magnetostatic and kinetic energy. On the other hand, at scales $\sim \lambda$ one can find the distribution of $\vec{B}$ and $\vec{H}$ from the London equation at constant $\vec{M}$. This is shown for the Bloch domain wall (the magnetization $\vec{M}$ rotates in the plane of the wall and does not produce the magnetostatic charges) in Fig. 4a. Though our picture corresponds to a 180° wall, a similar picture is expected for any domain wall. The jump of the tangential component of the moment $\vec{M}$ at the wall defines the current sheet, responsible for a jump of the magnetic induction parallel to the wall, whereas a possible jump of the normal component of $\vec{M}$ (a "charged" domain wall) would produce a jump of the normal component of the field $\vec{H}$.

The magnetic flux on the opposite sides from the domain wall creates the magnetostatic fields outside the sample, where the wall meets the sample boundary (Fig. 4b). The magnetic fluxes, which exit from the sample at two sides from the wall, are equal in magnitude ($4\pi M\lambda$ per unit length along the wall) but opposite in direction. The magnetostatic field from domain walls could be used for their experimental detection. At distances $r \gg \lambda$ from a line, where the wall exits to the sample boundary, this field is a dipole field of the order of $M\lambda^2/r^2$.

In summary, the letter presents the first analysis of the domain structure in superconducting ferromagnets. There is no equilibrium domain structure in a superconducting ferromagnet in the Meissner state. In the spontaneous vortex phase the period of the domain structure may essentially exceed that in the normal ferromagnet. But metastable domain walls can exist even in the Meissner state. They generate the magnetic flux in layers of a thickness λ, which can be revealed by magneto-optical methodic.

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[1] I. Felner, U. Asaf, Y. Levi, and O. Milo, Phys. Rev. B **55**, 3374 (1997).

[2] M.R. Eskildsen *et al.*, Nature **393**, 242 (1998).

[3] D. Aoki *et al.*, Nature **413**, 613 (2001).

[4] C.W. Chu *et al.*, Journ. of Superconductivity **13**, 679 (2000).

[5] C. Bernard, J.L. Tallon, E. Brucher, and R.K. Kremer, Phys. Rev. B **61**, R14960 (2000).

[6] H.S. Greenside, E.I. Blount, and C.M. Varma, Phys. Rev. Lett. **46**, 49 (1981).

[7] E.B. Sonin and I. Felner, Phys. Rev. B **57**, R14000 (1998).

[8] L.N. Bulaevskii, A.I. Buzdin, M.L. Kulic, and S.V. Panyukov, Adv. Phys. **34**, 176 (1985).

[9] L.D. Landau and E.M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1984).

[10] E.B. Sonin, cond-mat/0102102.

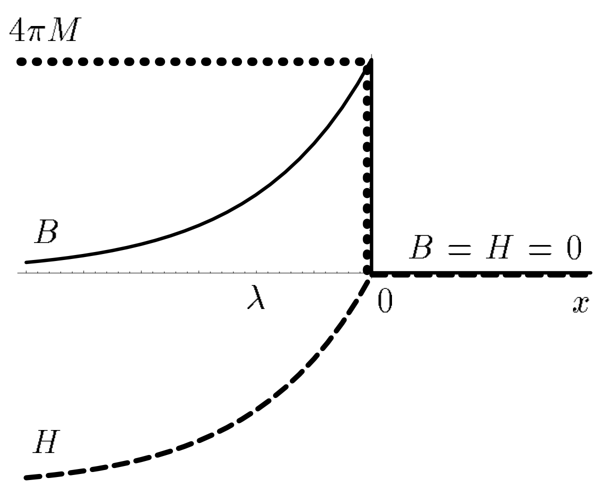

FIG. 1. Magnetic induction B (solid line), magnetic field H (dashed line), and $4\pi M$ (dotted line) at the boundary between a superconducting ferromagnet ($x < 0$) and vacuum ($x > 0$).

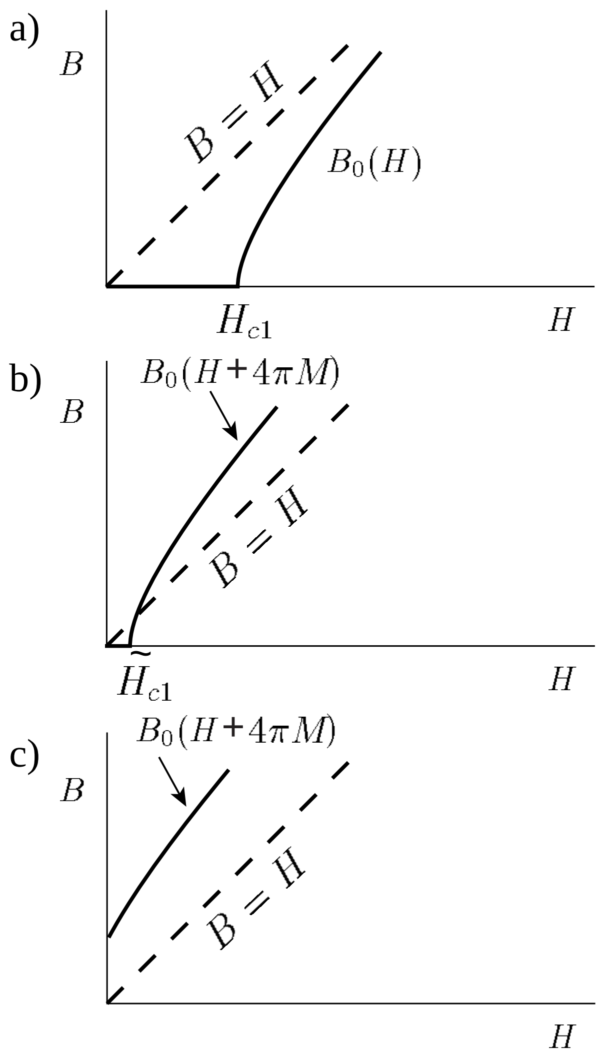

FIG. 2. Magnetization curve: a) nonmagnetic type-II superconductor; b) superconducting ferromagnet, $4\pi M < H_{c1}$; c) superconducting ferromagnet, $4\pi M > H_{c1}$.

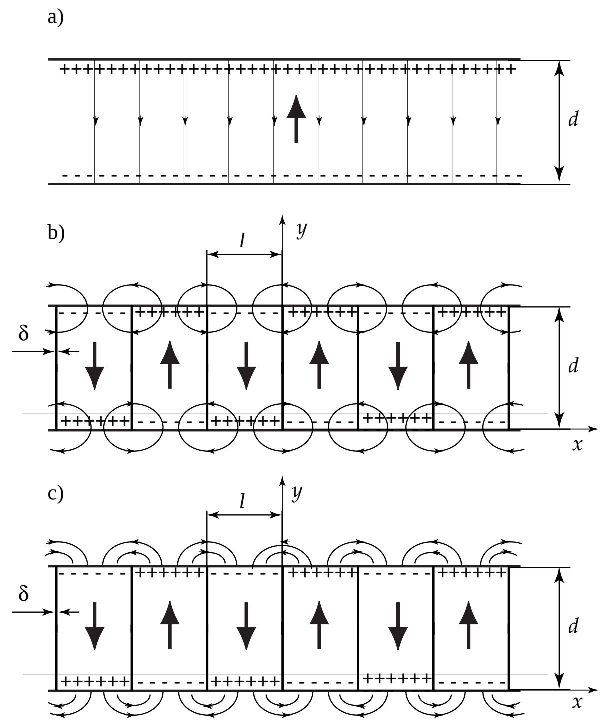

FIG. 3. Domain structure in normal and superconducting ferromagnets. The thick arrows show directions of the magnetic moment $\vec{M}$, the thin lines with arrows are force lines of the magnetostatic field $\vec{H}$. The magnetic charges are shown by + and -. a) A single-domain structure. In the whole bulk $B = 0$ and $\vec{H} = -4\pi\vec{M}$. b) A stripe domain structure in a normal ferromagnet. The magnetostatic fields are present in areas $\sim l^2$ inside and outside the sample. In the rest parts of domains $H = 0$ and $\vec{B} = 4\pi\vec{M}$. c) A superconducting ferromagnet in the spontaneous vortex phase with a rigid vortex array. The magnetostatic fields appear only in areas $\sim l^2$ outside the sample. In the bulk of domains $H = 0$ and $B = B_0(4\pi M)$.

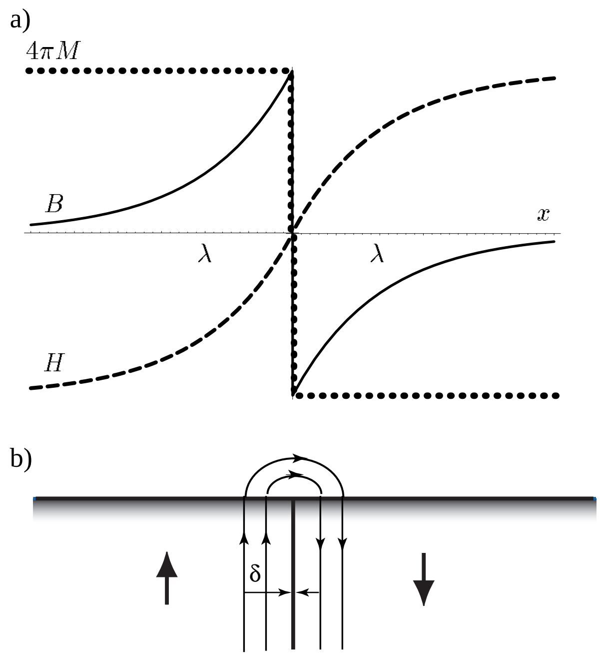

FIG. 4. Domain wall in the Meissner state: a) Magnetic induction B (solid line), magnetic field H (dashed line), and $4\pi M$ (dotted line) near the domain wall in the superconducting ferromagnet. b) Magnetic flux lines around the exit of the domain wall (of thickness δ) to the sample surface.